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## The Fourth Figure in Aristotle<sup>\*</sup>

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Research Article

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**Abstract:** This paper investigates whether Aristotle was aware of the fourth figure, and if he was aware of the fourth figure, why he excluded it from his system. Various commentators have explained why this figure does not exist in the system, so this paper compiles and examines these arguments through a certain logical frame. By inquiring into why the fourth figure was not included in his logical system, the paper considers whether logical factors may explain this exclusion.

**Keywords:** Aristotle, Aristotelian logic, logical frame, syllogism, the fourth figure.

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## Introduction

In order to construct a logical syllogism,

(1) there must be three terms,

(2) two premises must be constructed with these terms,

(3) one of the terms must be stated in both premises, but not in the conclusion.

Below are all syllogistic figure combinations with the terms M, S and P.

M P	P M	M P	P M
<u>S M</u>	<u>S M</u>	<u>M S</u>	<u>M S</u>
S P	S P	S P	S P
I. Figure	II. Figure	III. Figure	IV. Figure
P M	M P	P M	M P
<u>M S</u>	<u>M S</u>	<u>S M</u>	<u>S M</u>
P S	P S	P S	P S
V. Figure	VI. Figure	VII. Figure	VIII. Figure

Textbooks usually state that there are 256 forms of these initial four figures. Most logicians claim that twenty-four of them are valid, while some others say that nineteen of them are valid, and the rest assert that only fifteen are valid. Among the twenty-four forms are more commonly thought to be valid. By reversing the premises' order, we can find forty-eight valid forms.<sup>1</sup> The below table shows the twenty-four valid forms with their traditional mnemonic names:

<sup>1</sup> Colwyn Williamson, "How Many Syllogisms are There?" *History and Philosophy of Logic* 9 (2018), 77-85.

In The First Figure	In The Second Figure	In The Third Figure	In The Fourth Figure
MaP & SaM SaP (Barbara)	PeM & SaM SeP (Cesare)	MaP & MaS SiP (Darapti)	PaM & MaS SiP (Bramantip)
MeP & SaM SeP (Celarent)	PaM & SeM SeP (Camestres)	MeP & MaS SoP (Felapton)	PaM & MeS SeP (Camenes)
MaP & SiM SiP (Darii)	PeM & SiM SoP (Festino)	MaP & MiS SiP (Datisi)	PiM & MaS SiP (Dimaris)
MeP & SiM SoP (Ferio)	PaM & SoM SoP (Baroco)	MiP & MaS SiP (Disamis)	PeM & MaS SoP (Fesapo)
		MeP & MiS SoP (Ferison)	PeM & MiS SoP (Fresison)
		MoP & MaS SoP (Bocardo)	
Weakened Moods: Barbari, Celaront, Cesaro, Camestros, Camenop			

Aristotle's analysis of syllogisms shows that the first three figures exist in *Analytica Priora*. The question is, then, why Aristotle did not also include the fourth figure and its valid forms.

### A Brief History of the Fourth Figure

The oldest source who mentions the fourth figure is Theophrastus, who was Aristotle's student and successor. It is peculiar that Aristotle himself does not evaluate this figure within his system. Yet, we do not find any sources that consider the fourth figure as a separate figure for more than a thousand years after Aristotle's death. According to Hubien, the first thinker to consider this figure separately was Jean Buridan (fl. ca. 1300).<sup>2</sup>

<sup>2</sup> Hubert Hubien, "Jean Buridan on the Fourth Figure of the Syllogism," *Revue Internationale de Philosophie* 29 (1975), 271.

According to most history of logic books, this figure was first added to the Aristotelian understanding of syllogism by Galen of Pergamon (fl. ca. 129), hence this figure is called the Galenian Figure.<sup>3</sup> Yet, in both Galen's and his contemporaries' works, we find neither any explanation of this form nor any mention of its existence. Some commentators, such as Friedrich Ueberweg, Heinrich Scholz, J. W. Stakelum and Jan Łukasiewicz, asserted that there is no evident connection between the fourth figure and Galen.<sup>4</sup>

In *Institutio Logica* Chapters IX-XI, Galen mentions that particular affirmative conclusions can be achieved by premises conversion, because the universal affirmative conclusion can also be expressed as a particular affirmation. Particular negation conclusions can only be achieved from a universal negation conclusion but not by premises conversion. Galen refers to obtaining new forms in the first, second and third figures but does not refer to a new kind of figure.<sup>5</sup> Kieffer, the translator of Galen's work, comments that this is not a new method; an indirect way of obtaining this kind of imperfect syllogism can be found in Aristotle and Theophrastus, so Kieffer remarks, "There is no justification for attributing the invention of the fourth figure to Galen".<sup>6</sup> It is only talk that Galen himself accepts the fourth figure,<sup>7</sup> and he denies the existence of any figure other than the first three figures in chapter 12.1: "These syllogisms are called

<sup>3</sup> Thomas Reid, *Analysis of Aristotle's Logic, with Remarks* (Edinburgh: William Creech, 1806), 57.

<sup>4</sup> Friedrich Ueberweg, *System der Logik und Geschichte der Logischen Lehren* (Bonn: Bei Adolph Marcus, 1865), 341; Heinrich Scholz, *Concise History of Logic* (New York: Philosophical Library, 1961), 38; James W. Stakelum, "Why 'Galenian Figure'?" *The New Scholasticism* 16 (1942), 289-96; Jan Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* (London: Oxford Clarendon Press, 1957), 39.

<sup>5</sup> Galen, *Galen's Institutio Logica*, trans. John Spangler Kieffer (Baltimore: Johns Hopkins Press), 40-43.

<sup>6</sup> Galen, *Galen's Institutio Logica*, 102.

<sup>7</sup> Pamela Huby, *Theophrastus of Eresus. Sources for His Life, Writings, Thought and Influence: Commentary*, vol. 2, ed. & trans. William W. Fortenbaugh et al. (Leiden: Brill, 2007), 64.

categorical, as I have said, and it is not possible to construct them in more than the three mentioned figures or in any other number in each figure”.<sup>8</sup> Łukasiewicz quotes from Wallies in the introduction to his edition of Ammonius’ *On Aristotle’s Posterior Analytics*:

There are three kinds of syllogism: the categorical, the hypothetical, and the syllogism *κατά πρόσληψιν*. Of the categorical there are two kinds: the simple and the compound. Of the simple syllogism there are three kinds: the first, the second, and the third figure. Of the compound syllogism there are four kinds: the first, the second, the third, and the fourth figure. For Aristotle says that there are only three figures, because he looks at the simple syllogisms, consisting of three terms. Galen, however, says in his *Apodeictic* that there are four figures, because he looks at the compound syllogisms consisting of four terms, as he has found many such syllogisms in Plato’s dialogues.<sup>9</sup>

Łukasiewicz discusses this comment:

Galen divided syllogisms into four figures, but these were the compound syllogisms of four terms, not the simple syllogisms of Aristotle. The fourth figure of the Aristotelian syllogisms was invented by someone else, probably very late, perhaps not before the sixth century a.d. This unknown scholar must have heard something about the fourth figures of Galen, but he either did not understand them or did not have Galen’s text at hand.<sup>10</sup>

The first Arabic work is, source more light on the historical problems with Galen and the fourth figure, “On the fourth figure of the categorical syllogism, which is the figure attributed to Galen” by Najm al-Dīn Abū al-Futūḥ Aḥmad ibn Muḥammad ibn al-Sarī, often referred to as Ibn al-Ṣalāḥ.<sup>11</sup> He explains that:

<sup>8</sup> Galen, *Galen’s Institutio Logica*, 43

<sup>9</sup> Maximilianus Wallies, “Praefatio,” *Ammonii in Aristotelis Analyticorum Priorum Librum I Commentarium*, ed. Maximilianus Wallies (Berolini: Typis et Impensis Georgii Reimeri, 1899), ix.

<sup>10</sup> Łukasiewicz, *Aristotle’s Syllogistic*, 41.

<sup>11</sup> A. Ibrahim Sabra, “A Twelfth-Century Defence of the Fourth Figure of the

Ahmad ibn al-Tayyib al-Sarakhsi has related in his epitome of the *Analytica [Priora]* that someone (literally : *a man*) mentioned to his teacher Abū Yūsuf Yaḡūb ibn Ishāq al-Kindī that he had a Syriac 13 treatise (*maqālah*) of Galen on this topic (literally : *in this meaning*). But al-Kindī disavowed this [figure], and stated that a rational division requires only three figures and no others, and he has not acknowledged a fourth figure. And it has been related that [Abū Naṣr Muḡammad ibn Muḡammad ibn Tarkhān] al-Fārābī has a discussion (*kalām*) about the standing of this figure and its illegitimacy (literally: *its rejection*), which I have not seen. These, then, are the books which we have seen that have afforded discussion (or: *mention*) of this figure. As for the rest of the books and commentaries which have come down to us, those of Aristotle and Alexander and Porphyry and other ancients and moderns, we do not find them affording discussion (or: *mention*) of it, but all of them when they divide the figures I divide them into three, and stipulate that they have no fourth. And we have found Galen [himself] doing likewise in the ninth chapter (*maqālah*) of the *Peri Apodeixeiōs*, for he divided the assertoric (or: *categorical*) figures into three only and concluded with the statement that they have no fourth; and he does likewise in his *Book on the Enumeration of Syllogisms*, [But] we have not yet seen from among the books on logic [attributed to Galen] despite the great number of which the *Fihrist* speaks, any except for these two books. [But] there happened to come to us the discourse (*maqālah*) by a man known as Dinḡa the Priest (*Dinḡa al-qass*) entitled "The Fourth Figure of Galen".<sup>12</sup>

Interestingly enough, he quotes from some of the missing works of Galen. Rescher presented Islamic sources as evidence of the idea that this form belongs to Galen. For Rescher, there are two methods of obtaining figures: either we take two premises that have not been distinguished, in which case we obtain three figures, or we get a pair of premises that depend on a result,

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Syllogism," *Journal of the Warburg and Courtauld Institutes* 28 (1965), 15.

<sup>12</sup> Nicholas Rescher, *Galen and Syllogism* (Pennsylvania; University of Pittsburgh Press, 1966), 52.

from which we obtain four. Rescher holds that Galen may have examined these two different methods of producing figures separately in two books. It can be one of missing books of Galen's.<sup>13</sup>

The fourth figure was recognized by Peter of Mantua in 1483, and was debated by Peter Tartaret in 1480, by Richard Crackenthorpe in 1622, and by Antoine Arnauld in 1662.<sup>14</sup> Once we come to modern philosophy, we see many interpretations of Aristotle's figures:<sup>15</sup> that only first figure syllogisms can be valid (*Kant 1762*), that only two of the fourth figure syllogisms can be valid (*Maier 1900*), that none of the fourth figure syllogisms can be valid (*Prantl 1925*), that none of the syllogisms Aristotle raised in *Analytica Priora* 1.7 can be valid (*Maier 1900*), and that any valid syllogism will be rendered invalid if the order of its two premises is exchanged.

### **On the Existence of the Fourth Figure in Aristotle's Understanding of Syllogisms**

Aristotle divides forms of reasoning into the perfect (τέλειος) and the imperfect (ἀτελεῖς) ones: "I call perfect a deduction which needs nothing other than what has been stated to make the necessity evident; a deduction is imperfect if it needs either one or more things ..." (24b22-25).<sup>16</sup> According to Aristotle, only deductions in the first figure are perfect. In order to make the imperfect (in the second and third figures) forms perfect, they must to be reduced to the first figure: "it is clear too that all the imperfect deductions are made perfect by means of the first figure. For all are brought to a conclusion either probatively or *per impossibile*..." (29a30-33). Thereby the perfect syllogisms are the axi-

<sup>13</sup> Rescher, *Galen and Syllogism*, 20-1.

<sup>14</sup> Neil Tennant, "Aristotle's Syllogistic and Core Logic," *History and Philosophy of Logic* 35 (2014), 5, fn. 6.

<sup>15</sup> Marilyn Jager Adams, "Aristotle's Logic," *Psychology of Learning and Motivation*, vol. 18, ed. Gordon H. Bower (New York and Boston: Academic Press, 1984), 279.

<sup>16</sup> All quotations of Aristotle are from the English translation in *The Complete Works of Aristotle*, vol. 1, ed. Jonathan Barnes (Princeton: Princeton University Press, 2014).

oms of the syllogistic.<sup>17</sup> Ross argues that Aristotle's second and third figures can only be made perfect by reducing them to the first figure.<sup>18</sup> Also, Ibn Al-Sari shows that fourth figure forms can be reduced to the first figure by presenting these syllogisms:<sup>19</sup>

Bramantip: for PaM & MaS (By replacing the premises) MaS & PaM then PaS (from Barbara) SiP

Camenes: for PaM & MeS (By replacing the premises) MeS & PaM then PeS (from Celarent) SeP

Dimaris: for PiM & MaS (By replacing the premises) MaS & PiM then PiS (from Darii) SiP

Fresison: for PeM & MiS (By conversion of the premises) MeP & SiM then (from Ferio) SoP.

Fesapo: for PeM & MaS (By conversion of the premises) MeP & SiM then (from Ferio) SoP.

Avicenna (Abū 'Alī al-Ḥusayn ibn 'Abd Allāh ibn al-Hasan ibn 'Alī ibn Sīnā) states that there are two difficulties in reducing this figure, and Nasīrūddin Tūsī declares that both of them are the conversion of each two premises.<sup>20</sup>

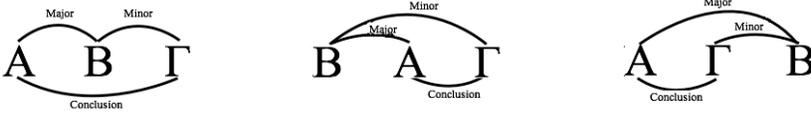
Rose states that when Aristotle gives a premise "AB" he means that A is predicated of B, which implies that A is the predicate and B is the subject. He claims that Aristotle established his syllogistic theory on this representation. In a syllogism with notation ABΓ, AB and BΓ are the premises and AΓ is the conclusion. Therefore, for the first figure, with ABΓ notation and AB as the major premise, BΓ is the minor premise and AΓ is the conclusion. For the second figure, with BAΓ notation and BΓ as the major premise, BA is the minor premise and AΓ is the conclusion. For the third figure, with AΓB notation and ΓB as the major premise, AB is the minor premise and AΓ is the conclusion;

<sup>17</sup> Łukasiewicz, *Aristotle's Syllogistic*, 43.

<sup>18</sup> W. David Ross, "Discovery of the Syllogism," *The Philosophical Review* 48 (1939), 251-72.

<sup>19</sup> Sabra, "A Twelfth-Century Defence of the Fourth Figure of the Syllogism," 21-6.

<sup>20</sup> Hüseyin Atay, "Mantıktaki Kıyasın Dördüncü Şekline Dair," *Ankara Üniversitesi İlahiyat Fakültesi Dergisi* 16 (1968), 36.



Rose further claims that Aristotle gives the major premise prior to the minor premise perhaps because of his symbolization. Yet we cannot make a similar triple representation for the fourth figure. Rose argues that since we cannot make this representation, the fourth figure was not present in Aristotle's system.<sup>21</sup> Without a possible symbolization, Aristotle did not include the fourth figure. If we assume that Aristotle remained loyal to this symbolization, one may wonder why he did not attempt  $\Gamma$ BA (which would ignore observance of the habit of reading from the left to right). However, Rose indicates that this situation violates a clear rule:

This circumstance is of course due to the fact that the major term is always written to the left of the minor term, with the result that the major premise is always in some way farther to the left than is the minor premise. Thus in each figure there is a natural and understandable tendency to state the major premise before the minor. But it is merely a matter of convenience, not yet the result of a rigid convention, and where the context is appropriate Aristotle has no qualms about stating the minor premise first.<sup>22</sup>

Additionally, in Aristotle's system, we do not see such symbolism or other research efforts; this kind of symbolization effort began after Aristotle. Rose supported this convention of writing the major premise first, Aristotle's choice of letters for the terms in each figure, and his failure to discuss the distribution of terms and the rules of the syllogism. Nevertheless, it is clear that this representation is suitable for Aristotelian syllogism.

Krois argues that Aristotle built a system of formal logic but that this system could not be understood until the 19<sup>th</sup> century. Aristotle's logical validity is only formal, yet it was not explained

<sup>21</sup> Lynn E. Rose, "Aristotle's Syllogistic and The Fourth Figure," *Mind* 74 (1965), 382-9.

<sup>22</sup> Rose, "Aristotle's Syllogistic and the Fourth Figure," 389.

in a formal way. Moreover, the validity of this formal structure does not bear any significance from Aristotle's perspective. For Koris, the fourth figure does not exist, and it does not have any importance for Aristotle's metaphysics.<sup>23</sup>

### Further Comments on the Fourth Figure

Varied commentators, noted below, claim that the reason Aristotle did not include the fourth figure is because it is irrelevant to his logic.

Henle argues that this problem is a psychological problem, rather than a logical one. Even though the fourth figure is not included, for Henle, he has built a perfect system.<sup>24</sup>

For Maritain, there is no place in logic for the fourth figure; it is just a grammatical debate.<sup>25</sup>

Türker claims that leaving out the fourth figure occurred due to Aristotle's inclination to make everything triple. He claims that this trilogy may be explained only by the methodical task of Hellenistic philosophy: there is no explanation apart from that. The only possible reason is then the importance of trichotomy.<sup>26</sup>

However, these and similar comments are outside the logical frame, I do not agree that.

### Tracing the Fourth Figure in Aristotle

As noted in the introduction, this paper's task is to show whether there is a fourth figure in Aristotle's syllogistic logic and whether Aristotle was aware of the figure. Hence, we must first look at how Aristotle explains figures. Aristotle explains the first figure in *Analytica Priora* chapter IV:

<sup>23</sup> John Michael Krois, "Validity in the Cultural Sciences?" *Discourse on a New Method: Reinvigorating the Marriage of History and Philosophy of Science*, eds. Mary Domski, & Michael Dickson, (Chicago and La Salle, IL: Open Court 2010).

<sup>24</sup> Paul Henle, "On the Fourth Figure of the Syllogism," *Philosophy of Science* 16 (1949), 94.

<sup>25</sup> Jacques Maritain, *An Introduction to Logic*, trans. Imelda Choquette (New York: Sheed & Ward, 1937), 187.

<sup>26</sup> Sadık Türker, *Batı Düşüncesinde Üçleme Sorunu* (İstanbul: Külliyat Yayınları, 2012), 147-8.

Whenever three terms are so related to one another that the last is in the middle as in a whole, and the middle is either in, or not in, the first as in a whole, the extremes must be related by a perfect deduction. I call that term middle which both is itself in another and contains another in itself: in position also this comes in the middle. By extremes I mean both that term which is itself in another and that in which another is contained. (25b32-37)

The second figure in chapter V:

Whenever the same thing belongs to all of one subject, and to none of another, or to all of each subject or to none of either, I call such a figure the second; by middle term in it I mean that which is predicated by both subjects, by extremes the terms of which this is said, by major extreme that which lies near the middle, by minor that which is further away from the middle. The middle term stands outside the extremes, and is first in position. (26b34-39)

The third figure in chapter VI:

But if one term belongs to all, and another to none, of a third, or if both belong to all, or to none, of it, I call such a figure the third; by middle term in it I mean that of which both are predicated, by extremes I mean the predicates, by the major extreme that which is further from the middle, by the minor that which is nearer to it. The middle term stands outside the extremes, and is last in position. (28a10-15)

Aristotle gives the description of the middle term after creating figures. Yet in a later part of *Analytica Priora* he explains all figures at once:

If then the middle term is a predicate and a subject of predication, or if it is a predicate, and something else is denied of it, we shall have the first figure; if it both is a predicate and is denied of something, the middle figure; if other things are predicated of it, or one is denied, the other predicated, the last figure. For it was thus that we found the middle term placed in each figure. (47a40-b6)

Peterson says that Aristotle's first figure is simple and structured stately; however, when other figures get involved, the sys-

tem becomes complicated and difficult to solve and understand. The fourth figure might lose attention in the system, that Aristotle established alone and without any help, due to the proliferation of figures. Further, due to the education approach of his school, the Lyceum, this figure might have remained undisclosed.<sup>27</sup> But, according to some, Aristotle determines the figures according to position of the middle term. We can see this explained when he writes

Since we know what sort of problem is established in each figure, and in which the universal and in what sort the particular is established, clearly we must not look for all the figures, but for that which is appropriate to the problem in hand. If it is established in more figures than one, we shall recognize the figure by the position of the middle term. (47b9-14)

Here Aristotle says that each figure should be based on the middle term while creating problems. This does not mean that he created figures based on the middle term. The figures already exist and which of these figures will be applied to the problem in question, the movement is based on the middle term. In a sense, this is a practical application of a theoretical structure. We see that he evaluates each permutation of syllogistic form according to the terms in the premise before. Therefore, the establishment of the figures would be completely a formal assessment; whether Aristotle was aware of a fourth figure is not under question.

Distinguishing between the orders of the two premises, known as the minor and major premises, started after Aristotle.<sup>28</sup> Thus, in determining the figures, the premises' position in the argument rather than their given terms is what matters. A description of the fourth figure made in a similar way would not be so different than the first figure.

In evaluating Aristotle's *Organon*, some commentators

<sup>27</sup> James B. Peterson, "The Forms of the Syllogism," *The Philosophical Review* 8 (1899), 371-2

<sup>28</sup> Charles H. Manekin, "Some Aspects of the Assertoric Syllogism in Medieval Hebrew Logic," *History and Philosophy of Logic* 17 (1996), 50.

(Łukasiewicz, Ross, Henle, etc.) find some matters missing, while others argue that some matters are treated in excess. According to Home and Kames, the majority view is that, in the *Organon*, the fourth figure is not involved in any way, neither accepted nor rejected; it is a matter that fails even to draw notice.<sup>29</sup> But some commentators, such as Ross, imply that Aristotle was aware of the forms that can occur in the fourth figure.<sup>30</sup> Although Aristotle does not mention the fourth figure, we can find the forms in the fourth figure indirectly. In 29a19-27, he recognizes that a universal or particular affirmation as a first premise and a universal negation as the second premise in the first figure yields as a particular negation conclusion, which amounts to recognizing the validity of Fesapo and Fresison in the fourth figure. Similarly, in 53a9-14 he recognizes the validity of the other moods of Bramantip, Dimaris, Camenes in the fourth figure.<sup>31</sup> See, for example,

e.g. if  $A$  belongs to every or some  $B$ , and  $B$  belongs to no  $C$ ; for if the propositions are converted it is necessary that  $C$  does not belong to some  $A$ . (29a23-25)

Forms in this passage are written in this way:

- 1)  $BaA \ \& \ CeB \ \therefore \ AoC$
- 2)  $BiA \ \& \ CeB \ \therefore \ AoC$

Once we change the location of the premises, we get:

- 1')  $CeB \ \& \ BaA \ \therefore \ AoC$  (Fesapo)
- 2')  $CeB \ \& \ BiA \ \therefore \ AoC$  (Fresison)

Patzig says that “it is clearly assumed that Aristotle saw the equivalence of (1) with (1') and of (2) with (2')”.<sup>32</sup> Additionally, Aristotle’s approach to (1) and (2) shows a conclusion in the eighth figure. Aristotle recognizes that with a minor term as  $C$  and major term as  $A$ , then in the conclusion, the minor term will

<sup>29</sup> Henry Home & Lord Kames, *Sketches of the History of Man*, ed. James A. Harris (Indianapolis: Liberty Fund, 2007), 665-6.

<sup>30</sup> Ross, *Aristoteles*, çev. Ahmet Arslan vd. (İstanbul: Kabcacı Yayınevi, 1995), 53.

<sup>31</sup> Ross, *Aristotle's Prior and Posterior Analytics* (London: Clarendon Press), 314.

<sup>32</sup> Günther Patzig, *Aristotle's Theory of the Syllogism* (Dordrecht: D. Reidel, 1969), 109-10.

be predicated to the major term. This stems from Aristotle's effort to incorporate this form with the first figure.

..., all the universal deductions give more than one result, and of particular deductions the affirmative yield more than one, the negative yield only the stated conclusion. For all propositions are convertible save only the particular negative; and the conclusion states one thing about another. Consequently, all other deductions yield more than one conclusion, e.g. if  $A$  has been proved to belong to every or to some  $B$ , then  $B$  must belong to some  $A$ ; and if  $A$  has been proved to belong to no  $B$ , then  $B$  belongs to no  $A$ . This is a different conclusion from the former. But if  $A$  does not belong to some  $B$ , it is not necessary that  $B$  should not belong to some  $A$ ; for it may belong to every  $A$ . (53a4-14)

Patzig stated this case by using a law of propositional logic,<sup>33</sup>

$$[(p \wedge q) \rightarrow r] \wedge (r \rightarrow s) \rightarrow ((p \wedge q) \rightarrow s),$$

which is called hypothetical syllogism.<sup>34</sup> With this expression we can get all weakened forms. From that we can derive the idea that Aristotle was aware of these forms. However, by changing the location of the premises we can acquire the other figures,

$$[(p \wedge q \rightarrow r) \wedge (r \rightarrow s)] \rightarrow (p \wedge q \rightarrow s) \rightarrow (q \wedge p \rightarrow s),$$

and by this expression, we can convert the forms of the first figure to the fourth figure,

MaP & SaM  $\therefore$  SaP (Barbara)  $\rightarrow$  MaP & SaM  $\therefore$  PiS  $\rightarrow$  SaM & MaP  
 $\therefore$  PiS (Bramantip)

MeP & SaM  $\therefore$  SeP (Celarent)  $\rightarrow$  MeP & SaM  $\therefore$  PeS  $\rightarrow$  SaM & MeP  
 $\therefore$  PeS (Camenes)

MaP & SiM  $\therefore$  SiP (Dariii)  $\rightarrow$  MaP & SiM  $\therefore$  PiS  $\rightarrow$  SiM & MaP  $\therefore$  PiS  
 (Dimaris)

<sup>33</sup> Patzig, *Aristotle's Theory of the Syllogism*, 111.

<sup>34</sup> Alfred N. Whitehead & Bertrand Russell, *Principia Mathematica*, Cambridge: Cambridge University Press, 1912, vol. 3, 112, prop. 3.33.

In this process, the results are syllogisms in the first figure, which are equipollent with the original. According to Henle:

Now Aristotle's theory of the syllogism bears every indication of being carefully worked out. All possible combinations of modal premises are considered and under each combination of modalities, the enumeration of different quantities and qualities of premises is fairly complete. Where there are omissions they can as a rule be supplied by the reader without much trouble. ... To summarize the situation with regard to Aristotle: the aim of his investigation is to discover conclusions of modal syllogisms. For this purpose, the fourth figure yields results easily obtainable otherwise and is not worth the trouble.<sup>35</sup>

However, this result is not as easily obtainable as Henle might think. Because of that, Peterson comments:

The moods of the fourth figure are nothing but varied forms of certain moods of the first and third figures and so we are restricted to the three figures recognized by Aristotle.<sup>36</sup>

The problem with this is that by converting the other figures to the fourth figure, the invalid forms become valid. For Merrill, all the valid and invalid forms that can be made in the first, the second and the third figure can also be converted into the fourth figure.<sup>37</sup> If we apply this to Datisi in the third figure, we should get a valid I-I-I form in fourth figure. Yet this form cannot be valid. In order to avoid this impasse, Peterson attempts to show the invalidity of some of the forms in the third figure. In my opinion, the problem with the fourth figure is that it has no place in an Aristotelian syllogism.<sup>38</sup>

With regards to Aristotle's theory, we may also make the same claim on the fifth, sixth, seventh and eighth figures, too. Additionally, we can be sure that Aristotle was aware of the other figures and the forms in these figures. The existence of the

<sup>35</sup> Henle, "On The Fourth Figure of the Syllogism," 102.

<sup>36</sup> Peterson, "The Forms of the Syllogism," 374.

<sup>37</sup> Daniel D. Merrill, "Reduction to the Fourth Figure," *Mind* 74 (1965), 66-70.

<sup>38</sup> Peterson, "The Forms of the Syllogism," 374-5.

fourth figure is not something that he missed;

If then we must take something common in relation to both, and this is possible in three ways (either by predicating *A* of *C*, and *C* of *B*, or *C* of both, or both of *C*), and these are the figures of which we have spoken, it is clear that every deduction must be made in one or other of these figures. (41a13-18)

### Assessment and Discussion

In my opinion, it is not failing or fallacy that Aristotle does not take seriously the fourth figure. His system is a successful one. However, it is an undue criticism to say that he was not aware of the existence of the fourth figure, or some other one. Clearly, he was well aware of the existence of the all other figures and deliberately did not take the fourth figure into account.

We have seen above that, if we give a definition of the fourth figure in public, practical use (e.g. by examples), its application is not quite different from the first figure. In this sense, if we agree on the idea that Aristotle's syllogism is not formal, we can also understand why he did not address this figure.

Nevertheless, I believe that Aristotle's system was a formal one. As we seen above, Aristotle has not failed to notice the existence of the fourth figure; on the contrary, he investigated it. But Aristotle's logic is a metaphysical system rather than one exclusively to be formalized. Aristotle did not consider predication as inclusion, as it is in the classical logic. According to Aristotle, this is a categorical arrangement.

I think that Rose's presentation above is compatible with Aristotle's system. However, I believe that Rose's explanation brings no clarity to the problem. Agreeing with Koris, I think that the problem originates for metaphysical reasons. Thinking differently about the existence of the fourth figure, I think that the reason for the absence of this figure can be explained by metaphysical rather than formal justification. Thus, it cannot be created in a different way than the other three forms. Thus, the question remains of why Aristotle gets this figure.

Scientific investigation is not about an object for Aristotle; it is about an incident or situation. It is not the case that, for a premise such as SaP, that S contains P. That is, subjects of the predicate S are subjects of the predicate P. So, for the fourth figure the mould that PM and MS, subjects of the predicate P are subjects of the predicate M and subjects of the predicate M are subject of the predicate S, in this case subjects of the predicate P are subject of the predicate S. This is eighth figure, not fourth.

First of all, it must be defined premise for to construct Aristotle's syllogism. Aristotle defines the simple statement as, "The simple statement is a significant spoken sound about whether something *"belongs to"* or *"does not belong to"* (in one of the divisions of time)" (17a22-24). Here, we define a premise, SP as whether what belongs to S also belongs to P or not. Thus,

$$\begin{array}{c} P \\ | \\ S \end{array}$$

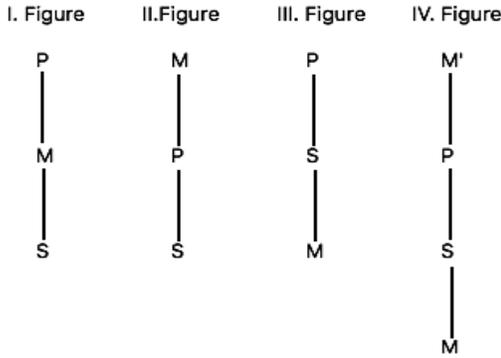
Aristotle gives how to construct a syllogism by premises,

For in general we stated that no deduction can establish the attribution of one thing to another, unless some middle term is taken, which is somehow related to each by way of predication... So we must take a middle term relating to both, which will connect the predications, if we are to have a deduction relating this to that. (41a2-13)

This passage's phrase, "each by way of predication," can be better understood as "by category". So we can show the syllogism as

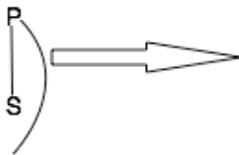
$$\begin{array}{c} P \\ | \\ M \\ | \\ S \end{array}$$

The other figures can be presented in the following way:



In this presentation, for fourth figure, M and M' are different terms. So we can see here the origin of Rose's comment. But if we want to understand why this figure has its conclusion, then we must engage in the following steps:

The other figures can be reduced to the first figure by conversion. Conversion is not a simple subject and predicate replacement. If a subject expands, then the subject will alter and the propositions will change from the first proposition. Hence the conversion would be:



The reduction of the second figure to the first figure is, then,



The idea that M is in between S and P comes from the first premise and we get its relation with S from conversing. The middle term is given in both premises and predicated to the other

terms. In this case, the first premise is reduced by conversing and then keeping the structure of the syllogism. As such, the third figure would be:



The idea that M is in between S and P comes from the first premise, and we get its relation with S from conversing. S is restricted, which protects the structure of the syllogism. Yet, in the fourth figure, in both premises it is not the case that there is an M in between S and P. If we try to get it, we will have this from:



M moved by this way will create a new term (because in this conversion, if the term extends, the term will be different and the premise will be too). In this case M will be a different cause in this syllogism than in the previous syllogism that is reduced to the first figure. In the second shape, the structure of the comparison with the expansion of P is disrupted, and thus is not applicable. In a similar way, if we disrupt the structure of the syllogism, and this is not applicable.

The form's middle term reduced from second and third figures to first figure maintains the structure. In the syllogism re-

duced from fourth figure, the middle term does not maintain the structure. Therefore, this syllogism's middle term is different.

We are deceived in such cases because something necessary results from what is assumed, since deduction also is necessary. But what is necessary is wider than deduction; for every deduction is necessary, but not everything which is necessary is a deduction. (47a31-34)

Every necessary conclusion is not a proof. Results obtained from the fourth figure are correct. However, the deduction is necessarily dependent on the middle term to come as a cause. The cause of the results obtained in the fourth figure is not then middle term in the syllogism.

In this syllogism, if we want to find as a result PS, it will be the first figure,



Yet this case is a syllogism of another inquiry. This inquiry is not about P, it is about S, i.e. first premise is MS and second premise is PM and conclusion is PS.

Now, we try to construct an example for fourth figure, for to clear my frame, with S: 'Animal', M: 'Human', P: 'Thinker';

Every Thinker is Human

Every Human is Animal

∴ Some Animal is Thinker

If we reduce to first figure by changing the propositions;

Every Human is Animal

Every Thinker is Human

∴ Some Thinker is Animal

Whereas, following is required to maintain syllogism form;

Every Human is Animal

Every Thinker is Human

∴ Every Thinker is Animal

Actually, the syllogism has been reduced in the first figure, converted by first premise

Every Human is Animal

Some Thinker is Human

∴ Some Thinker is Animal

In the second premise here, content of the middle term 'Human', is restricted, and considered as a new term. So not all of the category of 'Human' is examined; rather now a part of the 'Human' has been established for syllogism. In this case, the middle term in the last syllogism for 'Human' is different from the middle term of the first, it has been subset of the first. So, it is not possible to reduce Bramantip to Barbara, only to Darii.

I do not say here that the conclusion from the syllogism of Bramantip is invalid. Aristotle notes that other conclusions can be obtained by conversion of the other conclusion, as we have seen above. I intended to show; the fourth figure will not be established in Aristotle's system. Aristotle system is not to achieve the conclusion. I think that this entity has a structural purpose, which is the middle term. As we have seen here, the middle term for the conclusion obtained in the fourth figure can not be given as 'cause'. Here, it may come to mind, that this correlation applies to affirmation but not to denials. This is quite clear according to Aristotle. For Aristotle, the relation between the subject and the predicate is a predication. It is not the case that this predication is a positive or a negative one. We study this relation only after we make it. This distinction allows the structure shown in the referred figures to be understood in relation to each other. Reducing forms to each other is an entirely different matter. This is why, Aristotle mentioned the fourth figure, but did not take it as a syllogism. It cannot be built in Aristotle's logic.

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